



The Effects of heuristic teaching methods in mathematics

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Abstract

Article discusses the problem of heuristic teaching method of mathematics started from 19th centuries. Here was substantiated the thesis that the heuristic method of teaching is one of the most important methods of teaching mathematics. The works of scientists from different countries were studied. Based on the analysis of S.M. Jubran and other researchers, the level of necessity of this training method is established.

This article attempts to reveal the system of pedagogical skills of a teacher in heuristic teaching of mathematics. Here was shown pedagogical skills which are necessary for organizing and managing students' heuristic activities and fully describe the stages of the educational process (by O.A. Bulvinskaya).

In order to determine the conception of heuristic teaching method among teachers of Kazakhstan there was made a Likert scale questionnaire of 13 items. These 13 items were taken from 13 heuristics, that were identified by the Curriculum Planning and Developing Division (CPDD) within the Ministry of Education Singapore (MOE).

1. Importance of heuristic method of teaching

The term “heuristic” owes its origin to the legendary exclamation “Eureka!” (from Greek - found, discovered), with which Archimedes jumped out of the bath when he suddenly dawned on the solution of the task assigned to him by the ruler of Syracuse Hieron. The definition of heuristics in the shortest and clearest form sounds like "the science of how to make discoveries." This definition belongs to the outstanding mathematician and teacher of the twentieth century G. Polya. The origin of the heuristic apparently dates back to the time when there was formulated the question in ancient Greek philosophy: “How can we search for what we don’t know, and if we know what we are looking for, then why should we look for it?”

It is interesting to note that the prototype of the heuristic is considered mayeutics (Greek: midwifery, midwifery). This clever analogy is related to the concept of "Socratic conversations", which were a dialogue where there was no student and teacher - the interlocutors acted as equal partners in communication, for whom the truth and its knowledge were not given in a ready form. Achieving the right solution to the problem was done through a system of clever questions. After the interlocutor answered the first question put to him, Socrates asked the following additional questions in such a way that the partner's answers were in logical contradiction with the answer to the first question. Noticing contradictions, the interlocutor makes an amendment to the answer, which in turn was subjected to a new test by Socrates, or, as he called it, "rebuke", while revealing contradictions between the previous and the new answer. The interlocutor corrects the answer, and again a new contradiction arises before him. In this way, the student develops the belief that he had found a solution on his own.

In the following years, all the major teachers highly appreciated the Socratic method of achieving the truth in a dispute by overcoming contradictions in the opinions of the interlocutors. Thus, the great Czech teacher J.A. Comenius (1592 - 1670), protesting against the scholasticism and dogmatism of the medieval school, called for the revival of Socratic spiritual "midwifery art", stressed that to teach means "to discover hidden sources of knowledge in the mind", and not "to irrigate this source with other people's streams».

Modern pedagogy recognizes that the value of the Socratic method is not in getting answers to questions, but in designing the learner's heuristic activity in the process of dialogue.

German educator-democrat A. Diesterweg (1790-1866), who called the question-developing method of teaching of Socrates "the crown of the teaching art", wrote that "development and education can not be given or communicated to any person. Anyone who wants to join them must achieve this by his own activity, by his own strength, by his own effort".

Russian teacher and psychologist P.F. Kaptelev (1849 - 1922) also promoted the use of heuristic teaching in public schools and recommended that future teachers study it in teacher seminaries: "Heuristic form of teaching is one in which scientific laws, formulas, rules and truths are discovered and developed by the students themselves under the guidance of the teacher. The appearance of this form in the folk school question-answering (eritematoso). Questions are mostly suggestive". Recommending the teacher to use the Socratic method, P.F. Kaptelev advises "...compare items, find similar and different features... and group them into genera and species." The teacher, in his opinion, should guide them, "so that they do not stray from the straight path»

The outstanding Russian educator K.D. Ushinsky (1824 - 1870) was also an adherent of heuristics. "The best way to translate mechanical combinations into rational ones for all ages, and especially for children, "wrote K.D. Ushinsky" we consider the method used by Socrates and named after him Socratic. Socrates did not impose his thoughts on his listeners, but knowing what contradictory rows of thoughts and facts lay side by side in their dimly lit heads, he called these conflicting rows into the light circle of consciousness..." And so on: "If the teacher wants the child to clearly understand and really assimilate some new thought, it is best to achieve this in a Socratic way". At the same time, he stressed that the application of the Socratic method is not possible in all Sciences in the same way: it is most applicable in mathematics and philosophy, rather than in the Humanities or in history.

The formation of modern heuristics began in the mid-twentieth century. It was due to the development of cybernetics, which required the development of heuristic programming; the intensification of scientific, technical and inventive activities; the formation of new types of learning (problem, developmental, etc.) the fundamental research on pedagogical heuristics of this time should include the work of G. Polya ("How to solve it", "Mathematical discovery", etc.). G.Polya formulated and systematized the general rules underlying the search for solutions to problems. These rules were presented in the form of a certain sequence of questions and are still used in teaching mathematics. A system for organizing heuristic activity in solving problems, developed by G. Polya has shown high efficiency in stimulating the creative activity of students. Focused primarily on application in educational and cognitive activities, the heuristic system later became used in scientific and technical creativity.

Many studies in the field of didactic of mathematics are devoted to issues related to the use of heuristic approaches to teaching mathematics at all levels of schools (Prince, J.M., Felder, R.M. (2006), Kopka, J. (2007)). These studies demonstrate the necessity of using the activating teaching and learning methods and their considerable positive impact on affectivity of education. The use of this approach for teaching the pupils evidently improves their learning, knowledge and skills. Consequently, pupils are able to better understand the genesis of new concepts, their inclusion into a logical structure and the causes of their definition. These reasons lead to many recommendations to use heuristic approaches for teaching of mathematics much more than was practiced previously. Their presence can be useful to improve mastery of concepts and relations between them and a better understanding of the taught topic.

The positive results of heuristic teaching are seemed in S.M. Jubran's work:

The population of the study consisted of: All tenth grade students in Irbid the first educational directorate enrolling in the first semester 2011/2012. They form (3420) male and female students. The sample of the study comprised of (142) tenth grade students, 69 male and 73 female students in King Abdullah School at Irbid the first educational directorate and was distributed into four sections, which were selected purposefully. Two control groups and two experimental groups. The participants of the study were divided into two groups, experimental and control: The participants of the experimental group were taught by using heuristic approach for (8) weeks, While the participants of the control group were taught by using the conventional way for the same period.

The results showed that there was a statistically significant difference between the experimental group and the control group on the posttest, was

significantly better than that of the control group. To sum up, the researcher believes that the difference in the achievement of the tenth grade students was attributed to the using heuristic approach for teaching mathematics.

The one more study that was guided by Polya's (1957, 1973) belief that there are alternatives ways in solving a mathematical problem is the work of T.S.Hoon, K.L.Kee and P.Singh in Malaysia. The participants of this study raised a few comments in their reflective writings during the process of finding the right solution. For example,

“..I get the answer, based on the discussion with my group member. From my opinion, when we discussed in group, we can get more opinion to solve problem because every person has their own way on thinking and solve the question. Besides that, it is actually good working and discuss in group because it is better if we ask other's opinion before we solve it. We also can get a bit confidence when answering the question.”

“... I need to do many times until I get the correct answer. I refer to my friend on this question because I can't understand about the question. After that, we continue our class and discuss about ...”

2. The role of teachers' pedagogical abilities in heuristic teaching method

Among the many problems that are being solved today, our therefore, is the training of educated, mobile, competitive people who possess high moral qualities, intellectual and creative potential. In these conditions, one of the main tasks of the school is to form creative activity of schoolchildren, which can be provided by a

new methodological system – heuristic teaching of mathematics. However, the result of implementing this system depends on how much professionally, the teacher will be able to organize students ' search activities, form a creative personality, and manage the process development of methods of heuristic activity.

That is one of the most important tasks of the system of pedagogical education of future teachers and retraining of teachers is the formation of pedagogical skills of the teacher aimed at mastering the methodological system of heuristic teaching of mathematics.

The following pedagogical skills which are necessary for organizing and managing students 'heuristic activities fully describe the stages of the educational process. It is constructed adhering to the classification of O. A. Bulvinskaya.

Table 1.

Types of skills	Concept of skills
Design skills	set goals for heuristic learning in front of yourself and form them in students;
	design heuristic activities (choosing strategies and tactics) of students in the course of studying the topic;
	design lessons (or fragments of lessons) using heuristic techniques and heuristic learning tools;
	provide for possible difficulties in the organization heuristic activity of students;
	select didactic material focused on heuristic learning that corresponds to the school curriculum;
Constructive skills	find a variety of forms and methods of teaching that contribute to the organization of students ' heuristic

	activities;
	design technologies for heuristic teaching of mathematics;
	to make the heuristic tasks and the system of heuristic tasks;
	to make the heuristic guide to the performance of heuristic tasks;
Organizational and performing skills	to motivate the heuristic activity of students;
	call the student's readiness for independent discovery;
	to generate interest among students to explore topics through the organization of heuristic activity;
	encourage students to engage in creative activities related to expanding the use of heuristic activities through systems of heuristic and creative tasks;
	create and manage a heuristic situation in the lesson;
	use various heuristic techniques and training tools, including information and communication technologies;
	use general and special techniques of heuristic activity;
	to illustrate the heuristic and heuristic methods of solution mathematical problems;
	differentiated approach to students in the organization of their heuristic activities;
	reformulate a heuristic problem in different ways
	teach students the stages of solving heuristic problems;

Analytical skills	diagnose the level of preparation of students for heuristic activities;
	determine the level of formation of methods of heuristic activity in students;
	to provide timely assistance to students who have difficulties in the process of using heuristics;
	correlate the result of students ' heuristic activity with its goal;
	help students identify and understand the personal results of heuristic activity (reflection).

3. Heuristic teaching method in Kazakhstan and abroad

In many countries of the world, increasing attention is being paid to the problem of developing the creative abilities of schoolchildren at the present time. There are several methodical manuals and different books related with heuristic teaching method of mathematics. The greatest example is the book of G. Polya “How to solve it” (1945, Princeton University press, USA). It has been translated into several languages and has sold over a million copies, and has been continuously in print since its first publication. It shows how much this work is important and needed. But, unfortunately, there is no Kazakh version of this book. In 21st century this methodology still continues to be reflected in different books all over the world:

1. “Heuristics in problem solving for the teaching and learning mathematics” (2015, Portugal).
2. “Learning methods and techniques on how to solve problems” (2003, Germany).

3. “Explanation and proof in mathematics: philosophical and educational perspectives” (2009, Springer Science & Business Media).
4. “Teaching of mathematics” (New Delhi, 2008).

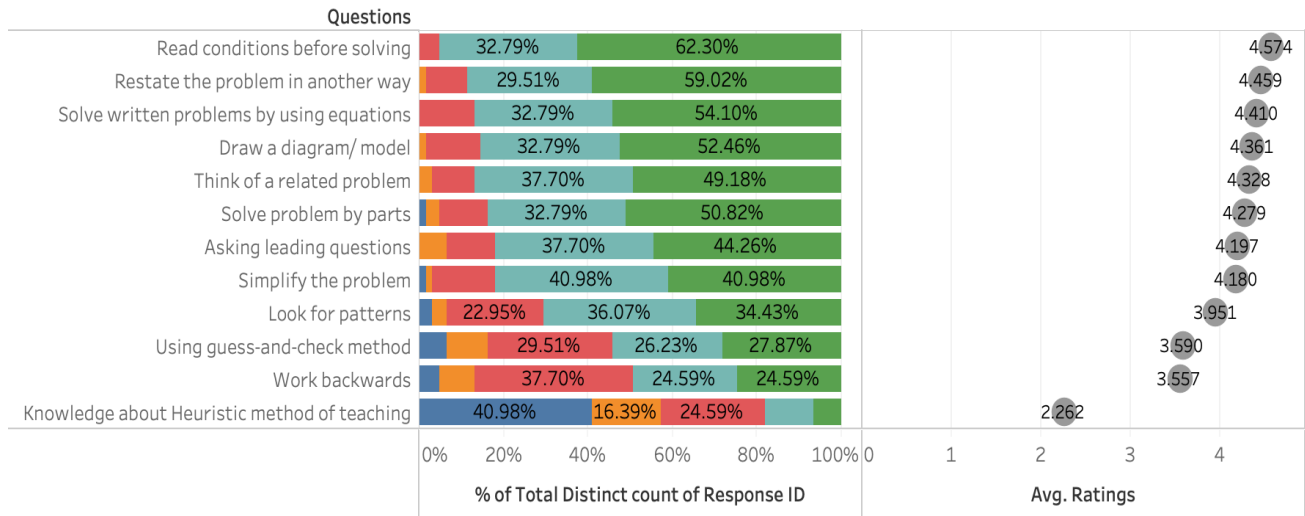
Some manuals were written as a result of research. “The book “Teaching of mathematics” is the result of an ardent experimentation and research of an author in making the problem solving in mathematics more effective and enjoyable.” Also the author states that “the work will be extremely useful to teachers, teacher-educators and student-teachers of mathematics.”

Another example for such a work is “Heuristics in problem solving for the teaching and learning mathematics” which is also a result of dissertation of Dr. A.F.Rodrigues. Due to the fact that thinking is cognitive, the author of this book introduces the reader to psychological concepts. Every teacher, even a math teacher, must be part psychologist. After the reader has figured out the concept of thinking, the author shows specific examples, problems in mathematics and explains how to solve them by heuristics.

In order to identify the level of heuristic teaching among teachers of Kazakhstan there was made a questionnaire of 13 questions. It was designed by the Likert scale in which 1 was strongly disagree, 2 disagree, 3 neutral, 4 agree and 5 strongly agree. Participants of this study were teachers from state educational institutions(12,3%) and educational centers (87,7%). The questions were taken from 13 heuristics, that were identified by the Curriculum Planning and Developing Division (CPDD) within the Ministry of Education Singapore (MOE). Also the same principles of heuristic teaching were designed by G. Polya.

The results are shown here below in Table 2.

Table



From this table we can see that the mostly used strategy by teachers is to teach students read the conditions of a problem before solving it. Its average shows 4.574 out of 5. Also, in order to better explain problems, that are difficult to understand, most teachers restate these problems in another way (88,53% of participants are generally agree with it).

Actually, there are no significant differences in application of these most teaching strategies like: solving problems by using equations, using diagram/model on lessons, teaching think of related problems if students don't have an idea about a given problem and teaching them to solve difficult problems by parts. This items show about 4.3-4.4 on average.

One of main ideas of teaching by heuristic method is asking leading questions to students in a process of solving a problem. From this analysis we can

see that this method of teaching is used quite well. Many teachers use it on their lessons (81.96%).

This table shows that teaching students to find an answer of a problem by looking for patterns is used rarely. Its result (3.951) especially stands out from other averages. And also very low results are shown in other two methods of teaching like “guess-and-check” and “work backwards”. Their results are about 3.6. Some teachers even disagree with a state that they are use it at all.

The last question was about knowledge of Heuristic method of teaching. The result was extremely low (2.262). From this we can state that almost all teachers don't know anything about such definition and concept.

This research indicates that elements of heuristic teaching are mostly used among mathematics teachers of Kazakhstan. Most average results are above 4.0 and it shows that teachers actually teach by heuristic method, but from the last result we see that they don't mind it and don't know about heuristics. In my opinion, it is better to explain pedagogues about such method of teaching in order to expand their concept about heuristics. The best manual for it is a book of G.Polya, “How to solve it”.

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