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Cardinality of survival set for the chaotic Tent map with holes

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Abstract

The aim of this paper is to give an overview of the dynamics of one dimensionaldiscrete dynamical systems: Tent map family T_3, Doubling map E_2 and shift map σ are investigated. Let I-intervals(Holes) lie in the interval [0,1) and let E_2 be a Doubling map. The survivor set Ω(I) :={x∈[0,1) : E_nx /∈ I, n≥0}. Depending on location and size of the intervals we will characterize the survivor set Ω(I) infinite or finite. Also we will show conjugacy of some maps that used in this paper. By using conjugacy of functions we will show that the Survivor set is infinite or finite in another composition of maps. The Cantor sets Λ that occur as non-survivor sets for c >2 from Tent map family T_c.[1]

Keywords: dynamical systems, symbolic dynamics, interval maps, survivor sets, chaos, open systems, irregular sets

Dynamics is the mechanism of time growth. It can be either deterministic, or sto chastic. Any devices also are difficult to forecast longer term. Only the normal geometry can't reflect its trajectories.Natural and social phenomenon also has unpredictability. Unpredictability is also an inherent attribute that is found at the very phenomena. It will have significant influence on human society and on science notion. Many problems remain throughout the human imagination, for example, how could a deterministic course be unpredictable? What are the reasons of symmetrical ice and snow flakes developing in Natural world? How does one call trajectories chaotic? May random be a deterministic trajectory? How can one characterize fluid motion and justify it? Would there be any order in core Chaos? How does one introduce chaotic dynamics to fractals of objects? There is no other direction we can address such concerns except studying nonlinear dynamics. The question of how one can symbolize a given dynamical system without losing any information of complexity in dynamics is one of the most intriguing subjects in analyzing informatio n processing in dynamical systems.

In general structure terms, Let (X, T) be dynamical system, where X is a compact metric space and T:X→X is positive topological entropy and continuous map. Let H be an open joined subset of X, named as a hole. The map T:X\H→X is called as open system since X\H may not be an invariant set coveredT.Let Ω(H) be the maximal T-invariant subset of X\H. Clearly

$$\Omega(H)=\{x\in X|T_n x \,/\!\in H, \text{ for every } n\geq 0.\}$$

In dynamical systems the survivor set mentioned as the set Ω(H).

The distinctive feature of open dynamical systems is that the orbits on X through H Hole, whereas in dynamical system (X,T) the orbits maintain their image on the space for every time n.

And here we listed some interesting and important research questions regarding th e system T|Ω(H) such as its cardinality properties. In our paper, more specific questions for H⊂X are what about conjugacy of functions and Is Ω_H (T) infinite? Our main research questions related to the survivor set Ω(H). For us interesting size of survivor set in open dynamical systems. We will find that survivor set is finite depending on location and size.

Definition(Topological conjugacy). Let $f : A \to A$ and $g : B \to B$ be two maps. f and g are topologically conjugate if there exists a homeomorphism $h:A \to B$ such that $h \circ f = g \circ h$. The homeomorphism h is called a topological conjugacy.

Doubling map. Consider the expanding map E_k: $[0,1) \to [0,1)$ with expansion constant $k \geq 2$ ($k \in N$) defined as $E_k(x) = kx \bmod 1$. For the doubling map (k= 2), Glendinning and Sidorov in [3] considered interval holes symmetric about the point 1/2, and asymmetric interval holes in [4]. In our case, We will consider expanding map with k= 2 (Doubling map). And we will use its canjugacy with shift map.

1.1. The chaotic tent map.

Yoshida T [2] analytically studied the behavior of the chatic tent diagram. In terms of invariant density and power spectrum, all over its chaotic field. Chaotic Tent Map is a regular and continuous map with a single fixed point. Once the goal height is that, consequent band-splitting transformations occur within the chaotic region and converge at the transfer stage into the nonchaotic field. The time-correlation function of nonperiodic orbits and their power distribution was specifically calculated at the band-splitting points and in the vicinity of other locations. The tent map is topologically conjugate, and so the actions of the map under iteration are in this way equivalent.

The chaotic tent map is given by:

$$T_c = \min\{cx, c(1-x)\}$$

Where $x \in [0,1]$. This map converts an interval [0,1] and includes only one c parameter for power. We will get this orbit for every x. Device 1.1 displays a variety of colorful actions, from repetitive to unpredictable, based on control parameter c. The picture of the turbulent tent diagram is seen when c=0.5,1,1.5,2 (Figure 1). Figure 2 demonstrates the structure of the family of tents for $c<1$. Interesting for us (Figure 3) if $c>2$.

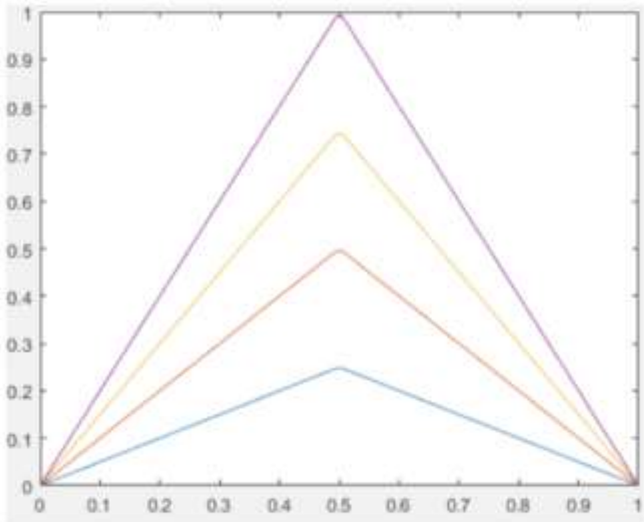

FIGURE 1. Graphs of the Tent map family when c=0.5,1,1.5,2.

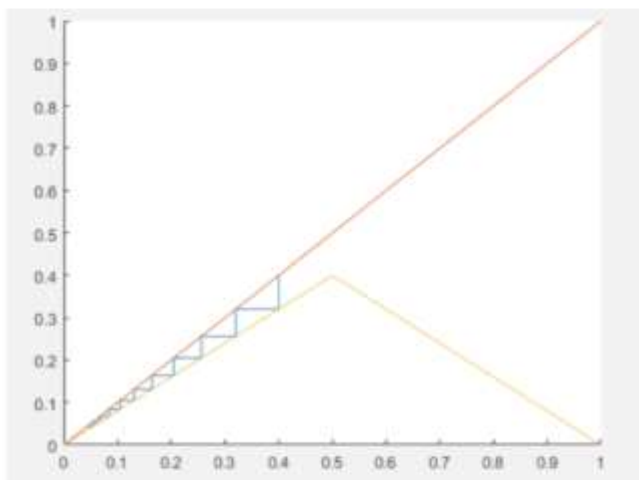

FIGURE 2. The dynamics of Tent map family for $c < 1$.

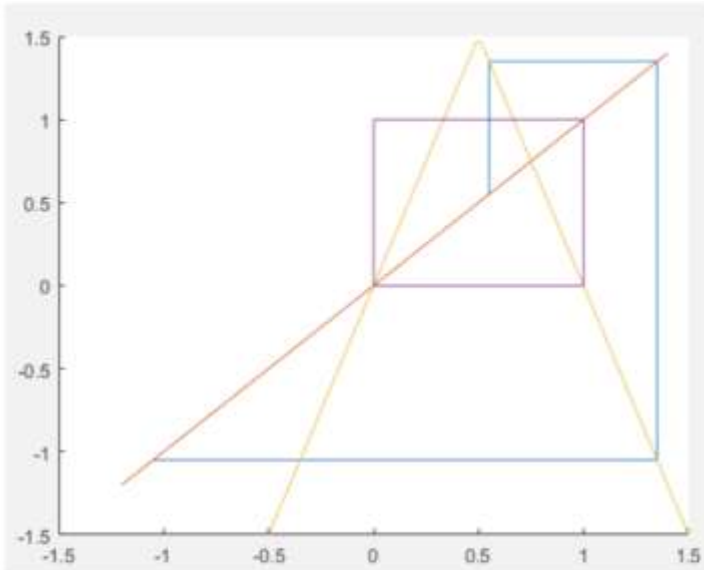

FIGURE 3. Dynamics of the Tent map family for $c > 2$. The purple square indicates the interval [0,1] and the horizontal T(x) = 0 and T(x) = 1.

1.2. Symbolic Dynamics.

Symbolic dynamics is a increasingly growing component of complex structures. Since it emerged as a tool for the analysis of generalized dynamic structures, the methods and concepts find major uses in database processing and storage and also linear algebra.

To abstract mechanics, shift spaces are what operations such as addition, subtraction, multiplication and division are to algebra. We begin by presenting such spaces, and explain a number of examples to direct the intuition of readers. We'll later reflect on different groups of move rooms. As the name may imply, there is a shift map of the change from the space to itself on each Shift space. Together such form a "dynamic shift space." They should concentrate on these dynamic structures, their relations and their implementations.

The sequences of symbols that we are researching are always constrained. The popular alphabet Morse is conveyed by a dot word and stitches with a total duration of at most six, such that a duration word of at least seven with no split is prohibited to happen (the one exception being the SOS signals). This ensures that the SOS signals are not used for all the symbols "line," "mark," and "stop." In programming language python, a code line such as cos(x)**2 ==y is not allowed, nor are unbalanced line axes, as Python's syntax laws ignore them. Different kinds of binary sequences have been used to fix major errors in compact audio discs, defined by a certain number of conditions In this segment we discuss the fundamental notion of shift space, which is a subset of points that follow constraints in complete shift.

For integer k >1, let Σ_k be the set of one-sided sequences with entries from the set Λ_k ={0,1,...,k-1}, excluding the sequences ending with (k−1)∞. For a finite length word w consisting of symbols 0,1,...,k−1,

we denote its length by |w|. Every such finite word w can be represented as ω0∞∈Σ_k. Set $B_k=\{\frac{l}{k^n} | l = 0,1,\dots,k^{n-1}, n\in N\}$. Let σ_k: $\Sigma_k \to \Sigma_k$ be the one-sided shift defined as

$$\sigma_k\,(a1,a2,a3,\dots) = (a2,a3,\dots).$$

We identify Σ_k with the interval [0,1) defined as $\pi_k\,(a1,a2,a3,\dots) = \sum_{n=0}^{\infty} \frac{a_n}{k^n}$, for (a1,a2,a3,...)∈ Σ_k. The map πk is a bijection, and the inverse image of any element of B_k is a sequence in Σ_k ending with 0∞. Representations of real numbers with an arbitrary base k > 1 . There $\pi_k^{-1}x\in\Sigma_k$ gives the k-expansion of x∈[0, 1). Note that the points (in B_k) have two k-expansions, one ending with 0∞, and other ending with (k−1)∞. It is well-known that the diagram given below (Figure 4) commutes. That is,$E_k\pi_k=\pi_k\sigma_k$, for all k≥2. A partial order ≺ can be defined on Σk as follows: u≺v if and only if either u1< v1, or there exists l≥2 such that ui= vi, for i=1,...,l−1, and $u_l < v_l$. For u,v∈Σ_k, we denote the set of all sequences w∈Σ_k such that u≺w and w≺v, including u and v, by [u, v], which is called an interval.

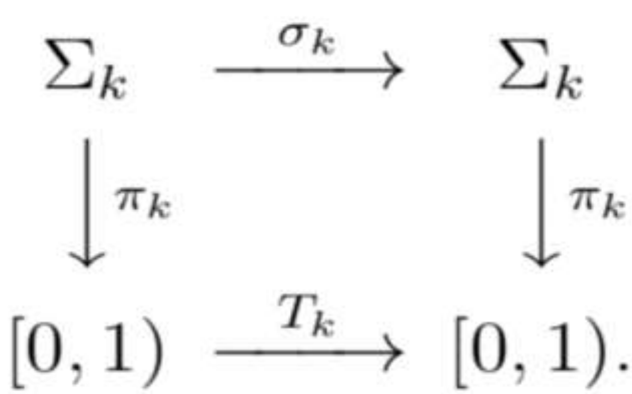

FIGURE 4. Commuting diagram.

1.3. Statement of results.

As it is seen from the literature that the previous work related to the survivor set was mainly to understand the cardinally of survivor set given the open dynamical system. We would like to extend these results with Tent map.

Lemma 1. Let function S: $\Sigma_2 \rightarrow [0,1]$ is defined as

$$S(a1,a2,a3,...)=\sum_{n=0}^{\infty}\frac{a_n}{3^n},.$$

Then there is conjugacy between T_3 and σ such that $S\circ\sigma=T_3\circ S$

Theorem 1. $\Omega(I_a)=\infty$ if and only if $\Omega(f(I_a))=\infty$.

2.Proof of main results

Proof of Lemma 1. We need to prove that S is bijection. We identify Σ_2 with the interval [0,1] defined as S(a1,a2,a3,...) $=\sum_{n=0}^{\infty}\frac{a_n}{3^n}$,,for (a1,a2,a3,...)$\in \Sigma_2$. The map S is a bijection, and the inverse image of any element of B_2 is a sequence in Σ_2 ending with 0∞. There $\pi_k^{-1}x\in\Sigma_2$ gives the k-expansion of $x\in[0, 1)$. Note that the points (in B_2) have two k-expansions, one ending with 0∞, and other ending with 1∞. It is well-known that is, $S\circ\sigma=T_3\circ S$.

Proof of Theorem 1. To prove this theorem we need to show that f:$\Omega(I_a)\rightarrow\Omega(f(I_a))$ is bijection. By lemma1 S is bijection and π_2 also bijection. It follows f=S$\circ$ π_k is bijection too. If f : A→B is bijection then |A|=|B|. f:$\Omega(I_a)\rightarrow\Omega(f(I_a))$. x$\in\Omega(I_a)$ and f(x)$\in\Omega(f(I_a))$ it follows x, E_2 (x),..., E_n 2(x)/$\in$ I_a $\Leftrightarrow$ x$\in \bigcap_{n=0}^{\infty} = E_2^{-n}(I_a)^c$. f(x)$\in$ $\bigcap_{n=0}^{\infty} =$ f$\circ$ $E_2^{-n}(I_a)^c = \bigcap_{n=0}^{\infty}$ f $\circ$ f^{-1} $\circ$ ${T_3}^{-1}$ $\circ$ $f(I_a)^c$=$\bigcap_{n=0}^{\infty} {T_3}^{-n}$$\circ$ $f(I_a)^c$ $\Leftrightarrow$ f (x) $\in\Omega$ (f (I_a)).

3.Conclusion

We studied open dynamical systems in tent maps and obtained that conjugacy between Doubling map and T_3 tent map. And we proved that if Survival set infinite in Doubling map then it is also infinite in T_3 tent map. Indeed, there is real interesting research question that what about real cardinality of survivor set. Is it depends on interval location and size? Another interesting question is to investigate the necessary condition for the survivor sets in T_k Tent map to be uncountable.

References

[1] van Harten, M. (2018). *The Dynamics of the One-Dimensional Tent Map Family and Quadratic Family* (Doctoral dissertation).

[2] Yoshida, T., Mori, H., & Shigematsu, H. (1983). Analytic study of chaos of the tent map: band structures, power spectra, and critical behaviors. *Journal of statistical physics*, *31*(2), 279-308.

[3] Glendinning, P., & Sidorov, N. (2001). Unique representations of real numbers in non-integer bases. *Mathematical Research Letters*, *8*(4), 535-543.

[4] Glendinning, P., & Sidorov, N. (2015). The doubling map with asymmetrical holes. *Ergodic Theory and Dynamical Systems*, *35*(4), 1208-1228.

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