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DIVERGENT TASKS AS A TOOL FOR PROMOTING MATHEMATICAL CREATIVE THINKING

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Abstract

Cultivating mathematical creativity is essential in mathematics education. One of the techniques for developing mathematical creative thinking is the implementation of divergent tasks. Through studying the contemporary literature this paper looks at various approaches to the definition of the concept of a divergent thinking and divergent problems. This article considers the possibilities of development of creative thinking of students on mathematics lessons through the use of the divergent tasks. Properties and the main types of divergent mathematical problems are distinguished. In addition several examples of divergent problems of different types are presented.

Keywords: creative thinking, divergent thinking, divergent tasks.

The concepts of divergent and convergent thinking were first considered in the works of the American psychologist J. Guilford (1967). Divergent thinking is a thinking that is characterized by the divergence of ideas to cover different aspects of the solution to a given problem or task (Razumnikova, O. M. 2013). Divergent thinking is considered as structural component of creative thinking (Tabach M., Levenson E., 2018). Convergent thinking refers to thinking that focuses on finding a single, distinct answer to a problem (Razumnikova, O. M. 2013).

Unfortunately, almost all of current teaching is aimed at actively developing convergent thinking, such a pedagogical bias does not develop, but rather suppresses the development of the creative person (Годфруа, Ж., 1992, ch.9). As is well known, Churchill and Einstein had difficulties in school. According to Godefroy(1992) “their teachers considered them undisciplined and absent-minded students, though that was far from it. The teachers were simply annoyed that they didn't answer the question directly, instead asking totally uncomfortable questions like "What if the triangle were reversed?", "What if water were replaced by ...?", "What if you looked at it from the other side", etc.”

Creative people are usually the ones who think divergently. They form new combinations from elements that the vast majority of people use in some particular way and they are able to form connections between elements which, at first glance, have nothing in common with each other (Tabach M., Levenson E., 2018). Let’s consider example purposed by Akhmedzhanov(1996), a task is given - to think of any drawing based on a circle. The first choices that come to mind are orange, sun, moon and face. These are standard answers, and they are what most respondents give. But sometimes there are completely unconventional answers, something like "a piece of Lambert cheese", "the footprint of an animal in round boots", or "a flat oasis against a desert background". Those kinds of answers are supposed to be creative. (Ахмеджанов, Э. Р., 1996) Even faced with domestic level problems, a person with divergent thinking supposes that there may be several options to address an issue; by contrast, for a person with convergent thinking any problem will be convergent. (Sharon Zmigrod et al., 2015) The development of divergent thinking is important not only for intellectual growth of a person, but also for his/her personal development. It cultivates such qualities of personality as tolerance, curiosity and, most importantly, creativity. (Sharon Zmigrod et al., 2015) That means divergent thinking is the framework for creative thinking.

Divergent tasks

In his book “The Nature of Human Intelligence” Joey Gilford (1967) defines a number of criteria that allow us to define the capacity for divergent thinking:

1. Fluency- refers to the number of ideas that arise in a unit of time;
2. Flexibility- the ability to switch rapidly from one idea to another, to see the unusual in insignificant details and find contradictions;
3. Originality- ability to think out of the box, to deviate from the set framework, established rules, elimination of template or stereotypical solutions;
4. Elaboration- using associations to express one's own ideas, working with symbols and images, finding complexity in simple things and simplicity in complex concepts.

In this regard, the terms "convergent tasks" and "divergent tasks" appear in the scientific literature (Bingölbali, E., & Bingölbali, F., 2020).

Traditionally in the process of mathematics teaching, convergent tasks are used as learning tasks, which mainly contribute to the development of convergent (logical) thinking (Tabach M., Levenson E., 2018). Convergent tasks assume the existence of only one (the only true) answer, which can be found through strict logical reasoning, based on the use of appropriate laws, rules, algorithms, formulas, theorems, etc. (Tabach M., Levenson E., 2018)

Divergent tasks, unlike convergent ones, are characterized by a variety of answers and solutions. The variability of ways of their solution and variety of answers creates favorable conditions for manifestation and development of the student's creative potential. Such tasks allow the pupil to put forward various hypotheses, ideas, conjectures, judgments, etc. and promote applying knowledge in new non-standard situations. (Silvia P. J. et al., 2008) Among other things, this develops the skill to critically approach the task text, to look at the given situation from all possible perspectives. Situations of varying degrees of uncertainty created by divergent tasks stimulate student activity (Mark A. Runco & Selcuk Acar, 2012). After all, in order to solve such tasks, a student needs to search for different approaches. To handle with divergent tasks, it is often necessary to use intuition, insight and other factors inherent in creative thinking. In this case students' thought processes act as catalysts, by training and developing creative potential. (Mark A. Runco & Selcuk Acar, 2012)

Divergent thinking exercises are often considered as providing opportunities for learners to show fluency in generating ideas, to demonstrate flexibility in the variability of their responses and to be creative in creating exceptional responses (Bingölbali, E., & Bingölbali, F., 2020).

Categorizing divergent tasks

One of the features of a divergent problem is to find different ways of solution, meanwhile conditions may be presented in different forms, and also several correct answers may exist. In his research, S.M. Krachkovsky's (2017, p.5) defined a divergent problem in mathematics a problem, which has at least one of the following properties:

- existence of different ways to solve the problem (it is essential that ways differ in idea, be original and do not be unreasonably tough in comparison with the others);
- presence of ambiguity in interpretations of a condition or requirement;

- possibility of interpreting the objects and situations in the task in several forms, through different models, integrating the task in different contexts.

Great German educator Diesterweg (1956, p.165) claimed that “it is more beneficial to study the same subject from ten different perspectives than to study ten subjects from one perspective”.

The existence of multiple solutions or even just two different solutions to the same mathematical problem is always an interesting, non-trivial fact that can give students' an additional incentive to study. Thus, the application of different types of divergent tasks in the process of teaching mathematics activates cognitive activity. It creates challenging situations and cognitive conflict, which increases interest in the study of the discipline and develops creative thinking of students. (Болдовская, Т. Е., & Девятерикова, М. В., 2020, 165)

A common type of divergent problems is those with uncertainty in the condition, which require consideration of several possible scenarios (Zaslavsky, O., 2005). Such problems can be found in geometry. Their peculiarity is that there may be several ways of geometric implementation of a given condition which most often leads to several answers. In such situation pupils may be involved in some exploratory activities. This includes searching for new "images" for the condition, checking if all possible constructions were considered, and justifying this. (Zaslavsky, O., 2005)

Even the well known formula is just assertion in strict defined wording and in another context it may have alternative message. Krachkovski(2016, p.99) gives a good example, the linear equation y=2x. “In a rectangular coordinate system, its graph is a line on the plane, in space it is a plane. If we take x and y as polar coordinates of points on the plane, then we can represent Archimedean spiral”.

The creative thinking is facilitated productively by looking for different ways of solving the same problem. On the one hand, solving problems in different ways is simply fun work, and on the other hand, such work forms the ability to see immediately, without solving the problem, that one or another way is not suitable for solving it, while another way will lead to a result.

Solving divergent problems students with different aptitudes have the opportunity to demonstrate their strengths (Grootenboer, P., 2009). For example, in class or as homework, everyone can be presented with the same problem and then have a discussion about how to solve it. In their way everyone can find their own method and see that it is not the only one, and that others may have different approaches to the problem and can achieve the same result, sometimes even in a more creative way. (Grootenboer, P., 2009)

In their article Gasharov and Makhmudov (2014) point some actions, that should be taken by students in order to solve divergent task. Those are:

1) Pupils use the matching method, where they analyse possible answers to the problem and exclude those that do not meet the conditions of the problem;

2) Pupils perform a variety of auxiliary models of the task;

3) Pupils consider different path of solving the problem;

4) The solution process relies on pupils' ingenuity, resourcefulness and life experience.

They (Gasharov and Makhmudov, 2014) also mention three types of divergent tasks: a) Tasks with missing data; b) Tasks for composing problems based on a given solution or equation; c) exercises for composition of numbers. O.V. Sviridenko (Saratov FIO Wiki, 2010) in her research gives wider categorization of the main types of divergent tasks. The list is given below.

1. Motion-related problems
2. Combinatorial problems
3. Tasks connected with a range of measurements
4. Geometry construction problems
5. Tasks connected with representation and construction of numbers
6. Optimization problems
7. Magic squares tasks
8. Composition problems based on a given solution or equation
9. Tasks with missing data
10. Forecasting tasks

The list of the main types of divergent tasks does not exhaust their diversity, but gives a definite idea of how they can be composed and used in mathematics teaching.

Divergent task examples

Specific examples of divergent tasks are given below:

Problem1. Angle A of 60° is given. Its bisector passes through the centre O of a circle with radius 4. The distance from the point O to the vertex of the angle is 10. Find the radius of the circle inscribed in the given angle and touching the given circle. (Крачковский С., 2017, p.31)

Solution purposed by Krachkovski(2017, p.31). This is a typical geometry problem whose condition satisfies several, in this case four, different configurations.

Figure 1(a) and 1(b) show, respectively, two variants for each, when given circle and the required circle touching each other externally and internally. The solution in each case is almost identical: we consider right triangles with the hypotenuse connecting the centers of the circles, one of the legs(cathetuses)is parallel to the side of the angle and the second leg(cathetuse) is perpendicular to this side. For example, in the triangle in Fig. 13 (a) the length of the hypotenuse is R+4, the length of the smaller leg is $\frac{R+4}{2}$, from another perspective it is R-5. So, we obtain: R =14. Answers: 2; 14; 6 or $4\frac{2}{3}$

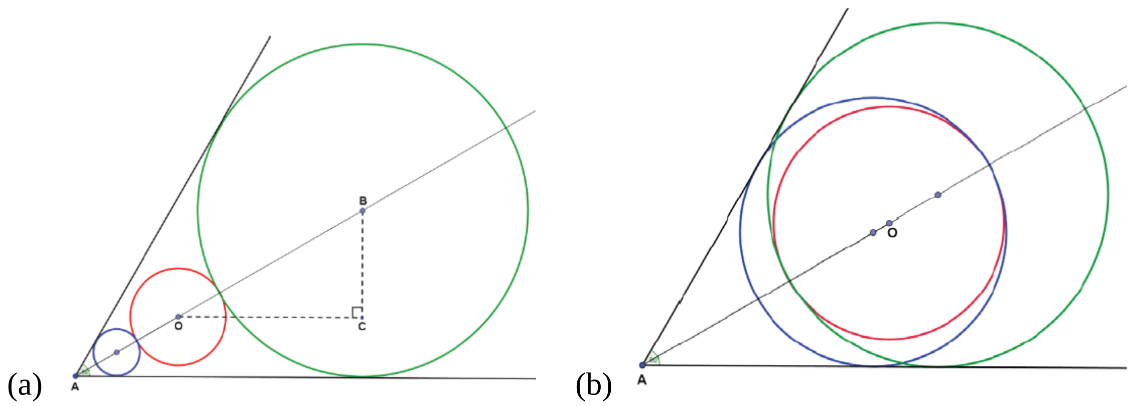

Figure 1 (Крачковский С., 2017, p.31)

Problem2. Prove that the midline of a trapezoid is parallel to the bases and its length is equal to their half-sum. (Трухманов, В. Б., & Трухманова, Е. Н., 2016).

This is a well-known theorem of elementary geometry. However, the task is not just to prove the theorem. It is necessary to find as many ways to prove the statement as possible.

Proof1. Let ABCD be the given trapezoid. E and F are the midpoints of the sides AB and CD, respectively, and K is the point of intersection of lines BC and AF (Fig. 2).

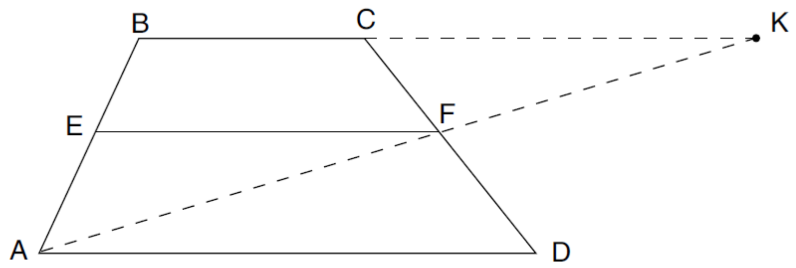

Figure 2. EF is the midline of the triangle ABK that underpins the proof of the theorem.

Proof2. In the trapezoid ABCD, M is the point of intersection of the diagonal AC and the midline EF (Fig. 3).

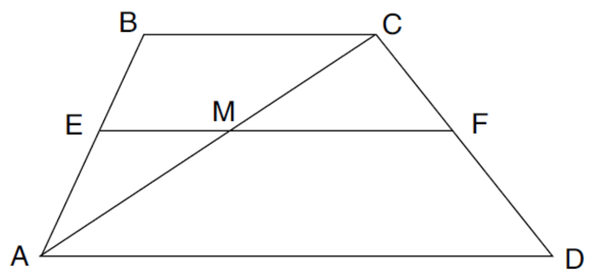

Figure 3. To prove the theorem, the property of the midline of a triangle is used.

Proof 3. Let's draw the line DN, that parallel to the side AB of the trapezoid ABCD (Fig. 4).

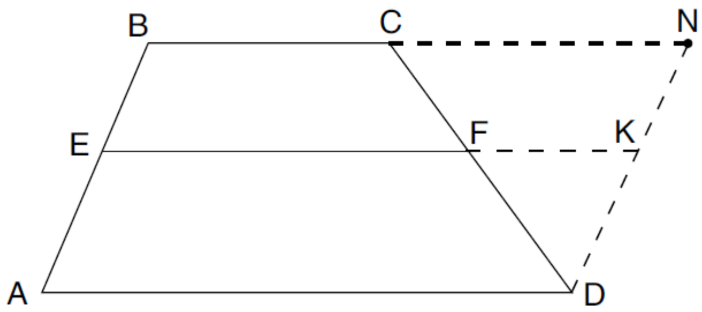

Figure 4

To prove the theorem, add the equations: EF+FK=AD and EF+FK = BC+CN, and use the property of the midline of the triangle DCN. After some simple manipulations we get the required statement.

Proof4. Draw the line KM parallel to the side AB in the trapezoid ABCD through point F, the midpoint of side CB (fig. 5).

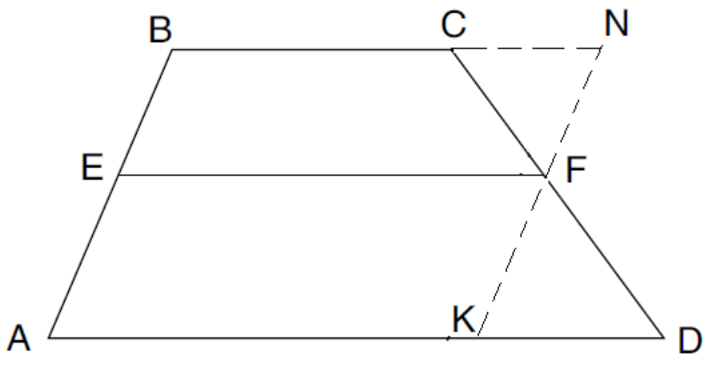

Figure 5

Just as in solution 3, add the equations: EF = AD-DK and EF= BC+CN. After a simple transformation and proving that CN = KD, we obtain the required statement.

Proof5. By tip to tail method $\overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AD} + \overrightarrow{DF}$ and $\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BC} + \overrightarrow{CF}$. After adding those two equations and simple transformations we obtain another proof using vector equality: $\overrightarrow{EF} = \frac{1}{2}(\overrightarrow{BC} + \overrightarrow{AD})$.

Problem3. Divide the square into four equal parts (Трухманов, В. Б., & Трухманова, Е. Н., 2016).

The division options are shown in figure 1. Children usually find the first three options easily. The other three do not rely on familiar, boilerplate solutions (Трухманов, В. Б., & Трухманова, Е. Н., 2016).

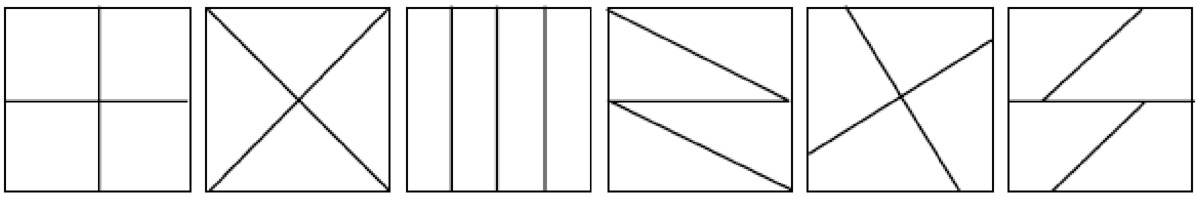

Figure 6

Problem 4. A two-digit number added to a mirror number gives a square number. Find out all possible numbers. (Далингер, В. А., 2014).

Solution. The two-digit number is$10a + b$. The mirror number is$10b + a$. The sum $(10a + b) + (10b + a) = 11(a + b)$ can be a whole square if $a + b = 11k$, where k is a square number. Since $a + b \leq 18$, then k = 1 and $a + b = 11$. (Далингер, В. А., 2014).

The answers are 29 and 92; 38 and 83; 47 and 74; 56 and 65.

Problem 5. Compare the numbers $\sqrt{2012} + \sqrt{2014}$ and $2\sqrt{2013}$. (Крачковский, С., 2017, p.19)

Solition1 purposed by Krachkovski (2017, p.19). Let's sketch a graph of the function $f(x) = \sqrt{x}$ and consider the trapezoid ABCD(fig. 7). Its bases have lengths $\sqrt{2012}$ and $\sqrt{2014}$, which means that the midline of the function is $MN = \frac{\sqrt{2012}+\sqrt{2014}}{2}$. Since the graph of the function is concave up on interval$D_f = [0; +\infty)$, the arc BC of the graph lies entirely above the chord BC, thereby the length of segment MC =$\sqrt{2013}$ is greater than the length of MN, and $\sqrt{2012} + \sqrt{2014} < 2\sqrt{2013}$.

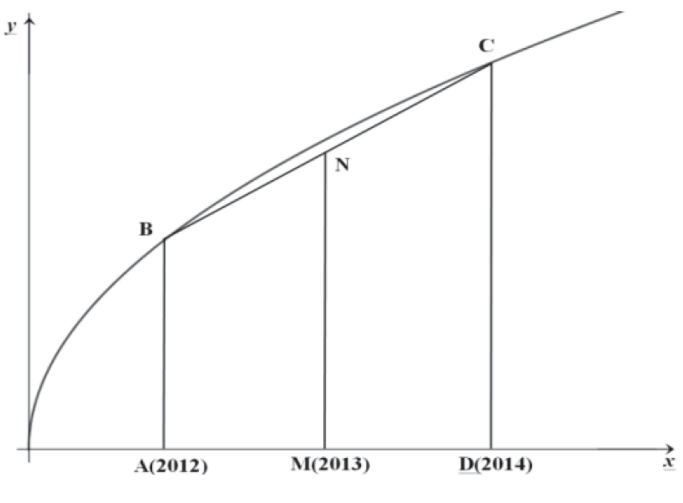

Figure 7

Despite its simplicity, such a solution is unfamiliar to most schoolchildren and, when they are introduced to it, it usually evokes interest and curiosity.

Solution2. Let's take k=2012, and compare $\sqrt{k}+\sqrt{k+2}$ and $2\sqrt{k+1}$. Than square both expressions, we obtain $2(k+1)+2\sqrt{k(k+2)}$ and $4(k+1)$ respectively. Then we just need compare $\sqrt{k(k+2)}$ and $k+1$, ensure, that $\sqrt{k(k+2)}<k+1$. So, we come to conclusion $\sqrt{2012}+\sqrt{2014}<2\sqrt{2013}$.

Problem 6. "From the cities A and B, the distance between which is 600 km, two trains left simultaneously towards each other. After 2 hours and 24 minutes the distance between them became 240 km. At what speed were they travelling, if the speed of the first train was 14 km/h greater than the speed of the other one?" (Крачковский, С., 2017, p.25)

Depending on, whether the meeting of trains has already taken place by the mentioned moment of time, two variants of the solution are possible and accordingly two answers: 82 and 68 km/h, or 182 and 168 km/h. Neither of them contradicts the stipulation. After all, trains can be freight and high-speed, and it was not specified in the condition. This is a simple problem for a grade 5-6. However, it is possible to show children the necessity of careful analysis of the data and writing down the complete solution, taking into account all cases and answers, satisfying the condition.

The answer is 82 km/h and 68 km/h, or 182 km/h and 168 km/h.

Problem7. The price of a product was changed twice by 20 percent. How many percent did the price change as a result? (Крачковский, С., 2017, p.27)

Solution. Depending on whether the price has risen or fallen for the first and second time, in general four situations are possible. Therefore there are three possible answers: increased by 44%, decreased by 4%, decreased by 36%.

Conclusion

The effectiveness of developing the creative thinking is very high, when using divergent tasks. Multiple variants of answers and solutions to tasks create favorable conditions for realization of the child's creative potential. It allows pupils to show fluency, flexibility, and originality of thinking in the process of working on task.

Application of divergent tasks in mathematics in the teaching process expresses considerable interest and great potential, both in terms of deepening knowledge of the subject area and in terms of developing students' creative thinking.

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