«SDU University»
BOOK OF ABSTARCTS AND PROGRAMME

International Emerging Scholars' Conference 2024:

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INTERNATIONAL EMERGING SCHOLARS CONFERENCE 2024

Towards Using Generative AI in Academic Research

3 June 2024

Program

Time	Activity		Place
9:00-10:00	**REGISTRATION / Welcome Coffee**		**SDU Life**
10:00-10:10	**Opening Ceremony** Welcome speech Alimzhan Igenbayev, Rector of SDU		**Blue Hall**
10:10-12:30	**Plenary session** *Moderator*: Maksat Kalybek, Vice-Rector for Research, SDU		**Blue Hall**
	10:15-10:45	Dr. Mariya Yesseleva-Pionka, SDU *"Scientific Writing and the use of AI"*	
	10:45-11:30	Dr. Shazim Ali Memon, NU *"Mastering Journal and Author metrics"*	
	11:30-12:00	Dr. Kairat Moldashev, SDU *"Using AI for data analysis: personal experience from several projects"*	
	12:00-12:30	Dr. Nursultan Kabylkas, NU *"Shedding Light To Verification Infrastructure Gaps with Generative AI models"*	
12:30-13:30	**Lunch break**		
13:30-15:00	**Sessions part 1 (simultaneous)**		
	Panel 1 (Computer Science and Engineering)		**Blue Hall**
	Panel 2 (Mathematics)		**205**
	Panel 3 (Education and Pedagogy)		**204**
	Panel 4 (Law and Social Sciences)		**201**
15:00-15:30	**Coffee Break**		
15:30-17:00	**Sessions part 2 (simultaneous)**		
	Panel 1 (Computer Science and Engineering)		**Blue Hall**
	Panel 2 (Mathematics)		**205**
	Panel 3 (Education and Pedagogy)		**204**

ABSTRACTS

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THE POSSIBLE EFFECTS OF GEOPOLITICS OF ARTIFICIAL INTELLIGENCE ON CENTRAL ASIAN AI-BUILDING

Artificial intelligence gained increased interest in recent years with the states involving this term in their national policy concepts and the development strategies. USA and China quite expectedly dominate the list of top countries engaged in AI development followed by other developed countries in Europe and Asia who increasingly utilize AI in military and economic purposes which in turn affect power distribution. However, few studies have evaluated the geopolitical projection of Central Asia involvement in AI as the factor for West-China, West-Russia competition. Being aware of the importance of AI, and being sandwiched between 2 major regional powers, namely Russia and China, Central Asia might become a playground of conflicting interests with the renewal of the West's dedication to stop authoritarian digitalization which had been observed earlier in Central Asia. Therefore, this paper aims to set a picture of possible regional dynamics in the area of AI development through conducting content analysis and incorporating case studies of Russia, China and West relationships with Central Asian states to identify the most probable set of scenarios in building local AI infrastructure. Presented work will discuss 2 hypotheses regarding the logic underneath the partnerships and policies CA states are implementing and might take into creation in the future. Firstly, existing institutional arrangements and traditional dynamics established between CA countries and major powers, especially those of China and Russia will probably become the primary tools in promoting export and regulation of AI technology. Secondly, CA states might have a diversified list of partners and channels in AI facilitation due to the prospective danger of technological dependence and public-private divide.

Key words: AI development, geopolitics of AI, power distribution in AI, Central Asian AI infrastructure, technological dependence

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THE POSITIVE EFFECTS OF SMOTE AND SELECTION OF OPTIMAL PARAMETERS WITH MLP ARCHITECTURE FOR IDS

This study proposes the positive effects of using SMOTE and choosing the optimal parameters with different activation functions of the backpropagation neural network model (BPNN) for the intrusion detection system (IDS). The proposed system was tested using comparative IDS datasets NSL-KDD and KDDCUP'99. The aim of this study is to achieve high detection accuracy, and low loss function with less iteration time. For this purpose, the optimal selections of MLP architecture and parameters (learning rate and momentum) of the backpropagation training algorithm using various activation functions for a number of neurons of both hidden and output layers were analyzed as one of the main problems of deep neural networks. Bi-polar sigmoid, Uni-polar sigmoid, Tanh, Conic Section and Radial Bases Function (RBF) as activation functions were used for experimental comparisons. Tanh function has been the best choice.

Key words: SMOTE, Intrusion Detection System (IDS), Back Propagation Neural Network (BPNN), NSL-KDD dataset, KDDCUP'99 dataset, Learning rate, Momentum,Activation functions

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A REINFORCEMENT LEARNING APPROACH FOR VEHICLE ROUTING PROBLEM WITH DRONES

Many exact algorithms, heuristics, and metaheuristics have been proposed to solve the Vehicle Routing Problem with Drones (VRPD), which involves using a fleet of trucks and drones to fulfill customer orders in last-mile delivery. In this study, we formulate this problem using the Markov Decision Process (MDP) and propose a Reinforcement Learning (RL) based solution. Our RL model is based on an attention-encoder and a recurrent neural network-decoder architecture. This approach enhances coordination by determining which vehicles should visit specific customers and where vehicles can rendezvous, effectively leveraging drones and reducing the overall completion time. Our proposed RL model has demonstrated competitive performance compared to benchmark algorithms through extensive experiments.

Key words: Vehicle Routing Problem with (VRPD), Markov Decision Process (MDP), Reinforcement Learning (RL).

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USING SVC AND LLM APPROACHES TO DETECT THE SQL INJECTION VULNERABILITY IN CODE

Injection attacks are a major concern ranking third in dangerous cybersecurity attacks. SQL Injection is one of the types responsible for 23 per cent of security vulnerabilities worldwide. In this research, we find an effective way to detect SQL Injection vulnerability in code using the Support Vector Classifier machine learning model and Large Language Model approaches. We obtained three datasets with 514 values. To compare the two approaches we used the numerical feature vectors process for the SVC model and turned the dataset into an acceptable format for the website OpenAI to work with the LLM model. The fine-tuning model based on gpt-3.5-turbo-1106 and the SVC model showed similar accuracy metrics of 96 per cent. The precision metric of the LLM model was higher. After comparing the models, both SVC and LLM models can achieve similar performance in detecting SQL Injection vulnerabilities. Future work aims to use the recent LLM models to reach higher performance.

Key words: SQL Injection, SVC, LLM, security, machine learning, AI

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USING AI TECHNOLOGIES IN CYBERSECURITY: ROBOTS AND ALGORITHMS

The rise in cybercrime necessitates the development of new protective measures. In 2023, Kazakhstan recorded 9,000 cybercrimes, a 13% increase compared to the previous

year. Artificial Intelligence (AI) plays a crucial role in automating cybersecurity tasks, including monitoring, analyzing, and responding to threats. Modern AI systems, based on robots and algorithms, can promptly identify and neutralize threats while analyzing vulnerabilities to minimize the impact of attacks. The study includes a systematic literature review and analysis of methods for enabling AI in cybersecurity strategy. Key areas of research include automation, threat detection and response, vulnerability analysis, protection against phishing and social engineering attacks, and countering AI attacks.

Keywords: cybersecurity, artificial intelligence, threat detection, vulnerability analysis, phishing prevention, social engineering, artificial intelligence attacks, ethical issues, legal framework.

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COMPREHENSIVE REVIEW ON VEHICLE ROUTING PROBLEM WITH TIME UNCERTAINTY: ADVANCES, CHALLENGES, AND FUTURE DIRECTIONS

This review paper provides an extensive analysis of recent developments in the Vehicle Routing Problem (VRP) under time uncertainty. It examines the challenges posed by dynamic and unpredictable factors in transportation logistics, such as variable travel times, fluctuating service durations, and inconsistent customer demands. By synthesizing research from various studies, this review categorizes and evaluates optimization algorithms, uncertainty modeling techniques, and simulation methods. Key topics include robust optimization strategies, dynamic dispatching models, and multi-objective optimization. The paper highlights the importance of addressing time uncertainties to improve route reliability and operational efficiency. Additionally, it identifies gaps in current research and proposes future directions for advancing VRP optimization in the face of temporal uncertainties. This review aims to serve as a comprehensive resource for researchers and practitioners, guiding them towards innovative solutions in vehicle routing under uncertainty.

Key words: Vehicle routing problem, time uncertainty, review, delivery problem

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REINFORCEMENT LEARNING BASED SWING UP CONTROL FOR INVERTED PENDULUM

Reinforcement learning (RL) has garnered considerable interest in modern industries due to its advancements in addressing challenging control tasks more effectively than rule-based programs. However, the robustness of RL remains an area under development, limiting its broader adoption. This issue is particularly pronounced as users increasingly turn to training simulations to reduce costs, creating a reality gap that adversely affects real-world performance. One common method to mitigate this problem involves randomizing uncertain parameters of the environment during training. However, this approach requires expert knowledge to determine the appropriate range of randomization. Another technique is fine-tuning agents by adapting their policies to new environments. However, it is challenging to adapt policies when the new environment requires them to be distributed differently than they were previously. These challenges restrict the practical use and popularity of both techniques. Our study proposes a hybrid approach to address these issues by fine-tuning agents trained with domain randomization through additional real-world training. To evaluate the effectiveness of our approach, we

conducted experiments with a rotary inverted pendulum, augmented with an extra weight not represented in the simulation. The results show that incorporating as few as twenty to fifty additional real-world training episodes can significantly enhance the performance of agents trained with domain randomization. Furthermore, adding fifty to two hundred additional episodes can elevate performance to a level comparable to agents fully trained in the real world. Our study concludes that efficient simulation-to-reality transfer is achievable with domain randomization and a relatively small amount of real-world training.

Keywords: Reinforcement learning Control, Inverted Pendulum, Deep Deterministic Policy Gradient

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ҚАЗАҚ ТІЛІН ШЕТ ТІЛ РЕТІНДЕ АШЫҚ БІЛІМ БЕРУ ПЛАТФОРМАСЫ АРҚЫЛЫ ҮЙРЕТУ

Қазіргі уақытта жоба жетекшісі Р.Абнасырова және жоба орындаушылары №AP19677903 «Қазақ тілін шет тіл ретінде оқытуда ашық білім беру платформасын пайдаланудың ғылыми-әдістемелік негіздері» атты мемлекеттік ғылыми жоба бойынша зерттеу жұмысын бастап кетті. Жобаның негізгі мақсаты елімізде білім алып жатқан және өзге мемлекетте жүрсе де қазақ тіліне деген қызығушылығы бар шетелдік азаматтардың мемлекеттік тілімізді үйренуге деген қызығушылығын арттыру үшін қазақ тілін шет тілі ретінде ашық білім беру платформасы арқылы оқытуды ғылыми-теориялық тұрғыдан негіздей отырып, қазақ тілін үйретудің қазіргі заман талабына сай әдістемесін ұсыну.

Түйін сөздер: Ашық білім беру, бейнесабақ, платформа, шетелдіктерге арналған қазақ тілі.

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ОБ ИСПОЛЬЗОВАНИИ ИИ В АУДИОВИЗУАЛЬНОМ ПЕРЕВОДЕ (СУБТИТРЫ)

Перевод субтитров или дублирование аудиовизуального материала для переводчика является одним из сложнейших и трудоемких процессов; не случайно в научной литературе утверждается, что аудиовизуальный перевод имеет междисциплинарный характер [1]. Однако в настоящее время существует большое количество нейросетей для работы в разных сферах, одной из которых является аудиовизуальный перевод. В нашей работе мы использовали искусственный интеллект для создания текстового формата аудио- или видео материала; в частности, для перевода субтитров мной были использованы два приложения. Одним из них является ClipChamp. Это приложение, в котором идет взаимодействие с искусственным интеллектом, в нем можно создавать субтитры из аудио- и видеофайлов. В свою очередь, ИИ форматирует аудио в текстовый формат; таким образом, переводчик может 80% процентов текста перевести с его помощью. Но имеются и некорректные моменты работы с ИИ: так, он иногда может не слышать некоторые слова или же не воспринимает акцент. Перед работой в данном приложении необходимо загрузить видео или аудио на компьютер; далее данный материал загружается приложение, где можно редактировать видео и его качество. Некоторые сложности создает формат субтитров (CPT), поэтому файл следует переформатировать в TXT-файл. Далее использовалось приложение Flixier, где редактируется видео- или аудиофайл. В данное приложение загружается видео, после чего идет работа с субтитрами из приложения Clip Champ. Преимуществом является то, что CPT формат автоматически создает тайминг,

соотносимый с тем или иным персонажем, и после загрузки файла приложение автоматически добавляет субтитры в нужные моменты; таким образом, около 70% работы переводчика осуществляет ИИ, но на долю переводчика приходится наиболее сложная ее часть, т.е. редактирование субтитров.

Ключевые слова: Искусственный интеллект, перевод, субтитры, аудиовизуальный перевод, ClipChamp ,Flixier.

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CREATING SUBTITLES FOR THE DOCUMENTARY "HOSTILE PLANET" WITH THE VIDEO EDITING PROGRAM CALLED DAVINCI RESOLVE

The aim of my project is to create high-quality subtitles from English into Russian, considering the background knowledge of the audience, cultural peculiarities. The practical significance of the study is the possibility of using the program in the sphere of audiovisual translation. The following translation transformations were used for translation into the target language: addition, omission, concretization, generalization, substitution, conversion, antonymic translation, holistic translation, explication, modulation. For instance, Fahrenheit was converted to Celsius because the subtitles need to be easy to read and instantly understandable to the viewer. It will take a while for viewers to mentally translate Fahrenheit into our temperature measurement. Moreover, There were numerous non-equivalent expressions found in the Russian translation, such as proper names and realias categorized by Barkhudarov. While working on the project, I encountered a wide range of challenges such as, identifying a suitable equivalent, comprehensible to the intended audience and editing the subtitles according to their requirements. Thus, in order to compose qualitative subtitles, it is not only significant to translate the content accurately, but also to think about their appropriateness. Despite some challenges, the translation and editing process was very engaging. As a result of the work accomplished in a practical way and the theory learned during the AVT course seminars, it was manageable to concisely and at the similar time correctly convey the connotative meaning of the narrative of the documentary.

Key words: subtitles, audiovisual translation, translation transformations, target audience.

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INTRODUCTION TO AUDIOVISUAL TRANSLATION TOOLS WITH A PRACTICAL APPLICATION EXAMPLE

The aim of this project is to identify and analyze the classic translation techniques used in audiovisual translation through a practical example of work from the Audiovisual Translation course. As it was stated by Chaume (2016), at the end of the 20th century, AVT “was named after the main practices encapsulated at that time: dubbing, subtitling, and voice-over” [1]. Over the course of one semester, a number of assignments such as subtitling, voice-overs, commercials, and analyzing the translation of completed film projects were completed in order to familiarize students with the basics of audiovisual translation and the practice of translating this type of content. In the process of studying this topic such methods were used as theoretical analysis of lectures provided by the course instructor for further application of the obtained information in practice; as well as comparative analysis in the context of comparing and analyzing already finished translations of various films and TV series. The study of this topic is aimed at the results

in the form of a deep understanding of the processes that take place in the field of audiovisual translation, in the understanding of the postulates of this direction in which future translators may be interested, as well as in summarizing specific techniques that can and should be used in audiovisual translation to create a quality final product, be it subtitles, commercials, or voice-overs.

Key words: Audiovisual translation, voiceover, subtitling, translation techniques

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ҚАЗАҚ ТІЛІН ШЕТ ТІЛІ РЕТІНДЕ МОБИЛЬДІ ҚОСЫМША АРҚЫЛЫ ҮЙРЕТУ

Тәуелсіздік алғаннан бері осы күнге дейін қазақ тілін шет тілі ретінде оқытудың практикалық тәжірибесі жинақталып, теориялық құнды еңбектер жарияланды. Қазіргі уақытта ол еңбектер қазақ тілін шет тілі ретінде оқытуда тиімді пайдаланылып жүр. Алайда жаңа ғасыр – цифрлық технологиялардың дамыған ғасыры. Цифрлық дәуір білім мен педагогтің жан жақты дамуын, жаңа заманға сай бейімделуін талап етеді. Цифрлы Қазақстанда жаңа ұрпақ жаңаша әдістерді қажет етеді. Әсіресе, тіл үйренуші, шетелдік азаматтар, цифрлық технологиядан біршама хабардар. Жаһандану заманында Қазақстанның халықаралық кеңістіктегі саяси, экономикалық беделіне сай өзге ұлт өкілдерінің қазақ тілін үйренуге деген қызығушылығы артуда. Шетел азаматтарының сол қызығушылығын жоймай, қазақ тілін оңай әрі жылдам үйренуі үшін біз тілдерді оқытудың жаңа заманға сай жаңа тәсілдерін іздеуіміз қажет. Елімізде білім алып жатқан және өзге мемлекетте жүрсе де қазақ тіліне деген қызығушылығы бар шетелдік азаматтардың мемлекеттік тілімізді үйренуге деген қызығушылығын арттыру үшін қазақ тілін шет тілі ретінде үйрететін мобильді қосымша дайындау арқылы өзге ұлт өкілдеріне қазақ тілін үйрету.

Түйін сөздер: IOS, мобильді қосымша, тіл саясаты, қазақ тілі.

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EXTENDED CAFFARELLI-KOHN-NIRENBERG INEQUALITIES WITH REMAINDER TERMS

In this work, our primary focus will be on remainder terms for the extended Caffarelli-Kohn-Nirenberg inequality from [1]. For this, we first obtain a weighted L^pHardy identity with weights for all complex-valued functions $fC0\infty(Rn \quad 0$) that implies an improved Hardy inequality. Moreover, we show cylindrical Hardy inequalities L^p for all complex-valued functions $fC0\infty(Rnx' = 0$). As a byproduct, we also discuss improved versions of the classical Caffarelli-Kohn-Nirenberg inequality and Heisenberg-Pauli-Weyl type uncertainty principles. In particular cases, these inequalities imply new functional inequalities that are not covered by the classical Caffarelli-Cohn-Nirenberg inequalities. Moreover, in some cases we show optimality of constants. In addition, we also discuss results on the setting of homogeneous Lie groups.This talk is based on the joint research with Nurgissa Yessirkegenov (SDU University, Kazakhstan).

Keywords: Caffarelli-Kohn-Nirenberg inequality, Hardy identity, anisotropic Euclidean space.

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THE COMPARISON PRINCIPLE FOR NONLINEAR PARABOLIC EQUATIONS WITH NONLOCAL SOURCE AND GRADIENT ABSORPTION AND ITS APPLICATIONS

In this talk we consider the following problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = \\ \quad \alpha|u|^{k-1}u \displaystyle\int_\Omega |u|^s dx - \beta|u|^{l-1}u|\nabla u|^q + \gamma u^m + \mu|\nabla u|^r \\ \quad - \nu|u|^{\sigma-1}u, \qquad x \in \Omega,\ t > 0 \\ u = 0, \qquad x \in \partial\Omega,\ t > 0 \\ u(x,0) = u_0(x), \quad x \in \Omega \end{cases} \tag{1}$$

where $\Omega \subset R^N$ ($N \geq 1$) is a smoothly bounded domain and $p > 1$, $\alpha > 0$, $\beta, \nu \geq 0$, $\gamma, \mu \in R$. In this talk we show a comparison principle for the problem (1). Moreover, we discuss its applications in studying global well-posedness and blow-up properties of weak solutions to the problem (1). Our research is motivated by the works [1], [2],[3] and [4].

Key words: parabolic equations, nonlocal source, gradient absorption.

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IMPROVED HYDROGEN UNCERTAINTY PRINCIPLE WITH MORE GENERAL WEIGHTS

Recall that the hydrogen uncertainty principle takes the form (see e.g. [1]-[2]):

$$\int_{\mathbb{R}^n} |u|^2 dx \int_{\mathbb{R}^n} |\nabla u|^2 dx \geq \frac{(n-1)^2}{4} \left(\int_{\mathbb{R}^n} \frac{|u|^2}{|x|} dx \right)^2, \tag{1}$$

for any function $u \in C_0^\infty(\mathbb{R}^n), n \in \mathbb{N}$, where the constant $\frac{(n-1)^2}{4}$ is known to be sharp. Moreover, it is known that the minimizers are of the form $u(x) = \alpha e^{-\beta|x|}$, $\beta > 0$, $\alpha \in \mathbb{R}$.

In this talk, we discuss Improved Hydrogen Uncertainty Principle with more general weights and for any homogeneous quasi norm which holds following form: Let $|\cdot|$ be any homogenous quasi-norm on $\mathbb{R}^n$. Let $\Omega \subseteq \mathbb{R}^n$, $V \in L^1_{loc}(\mathbb{R}^n)$ and ω be a positive function satisfying $-\left(\frac{d^2}{d|x|^2} + \frac{n-1}{|x|}\frac{d}{d|x|}\right)\omega + V\omega \geq 0$ in Ω. Then for all $u \in C_0^\infty(\Omega)$

$$\int_{\mathbb{R}^n} \left(\left| \frac{d}{d|x|} u \right|^2 + V|u|^2 \right) \mathrm{dx} \{ \geq \int_{\mathbb{R}^n} \left| \frac{d}{d|x|} (\omega^{-1}u) \right|^2 \omega^2 \mathrm{dx}. \tag{1}$$

Equality holds when $-\left(\frac{d^2}{d|x|^2} + \frac{n-1}{|x|}\frac{d}{d|x|}\right)\omega + V\omega = 0$.

Which contains Heisenberg Uncertainty Principles, Hydrogen Uncertainty Principles, Hardy Iequality and Linearized Sololev Inequality. Moreover, we obtain such type of uncertainty priciples with sharp constants and explicit extremizers. We also show more general versions of these results on homogeneous Lie groups. This talk is based on the joint research with Nurgissa Yessirkegenov (SDU University and Institute of Mathematics and Mathematical Modeling, Kazakhstan).

Keywords: Hydrogen uncertainty principles, Heisenberg Uncertainty Principles, Linearized Sololev Inequality, Hardy Iequality, sharp constant, extremizer.

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FACTORIZATIONS AND UNIFIED HARDY INEQUALITIES

In this note, we start by recalling the results from [1]:

$$\int_{\Omega} |(\partial_r f)(x)|^2 \, d^n x \geqslant \int_{\Omega} |x-x_0|^{-2} |f(x)|^2 \left\{ \frac{(n-2)^2}{4} + \frac{1}{4} \sum_{j=1}^{m} \prod_{k=1}^{j} \left[\ln_k (\gamma/|x-x_0|) \right]^{-2} \right\} d^n x, \quad (1)$$

valid for $f \in C_0^{\infty}(\Omega)$, assuming that $\Omega \subset \mathbb{R}^n, n \in \mathbb{N}, n \geqslant 2$, is open and bounded with $x_0 \in \Omega, m \in \mathbb{N}$, and the logarithmic terms $\ln_k (\gamma/|x-x_0|), k \in \mathbb{N}$, are recursively given by

$$\ln_1 (\gamma/|x-x_0|) := \ln (\gamma/|x-x_0|), \quad 0 < |x-x_0| < \gamma,$$
$$\ln_{k+1} (\gamma/|x-x_0|) := \ln (\ln_k (\gamma/|x-x_0|)), \quad 0 < |x-x_0| < \gamma/e_{k+1}, \quad k \in \mathbb{N},$$

for $\gamma > 0, x \in \mathbb{R}^n \setminus \{x_0\}, n \in \mathbb{N}, n \geqslant 2$, with $0 < |x-x_0| < \operatorname{diam}(\Omega) < \gamma/e_m$, where

$$e_1 := 1, \quad e_{k+1} := e^{e_k}, \quad k \in \mathbb{N}.$$

We denote $\sum_{j=1}^{0}(\cdot) := 0$ and $\prod_{k=1}^{0}(\cdot) := 1.$, so when $m = 0, x_0 = 0$.

In this talk, we discuss the inequality (1) with a more general weight, using the factorization method of differential operators from [1]-[3]. Moreover, we show the sharp remainder formula for the inequality (1).

Furthermore, we discuss the generalizations of these results on homogeneous Lie groups.

This talk is based on the joint research with Nurgissa Yessirkegenov (SDU University and Institute of Mathematics and Mathematical Modeling, Kazakhstan).

Keywords: factorization method, Hardy inequality, homogeneous Lie group, stratified group.

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HARDY AND RELLICH IDENTITIES RELATED TO BAOUENDI-GRUSHIN OPERATOR

Let us revisit the Hardy inequality related to Baouendi-Grushin operator by Garofalo [1]. Suppose z = (x1, . . . , xm, y1, . . . , yk) or simply z = (x, y) ∈ $R^m \times R^k$ with m + k = n and m, k ≥ 1. The sub-elliptic gradient is the n dimensional vector field given by

$$\nabla\gamma = (X1, \ldots, Xm, Y1, \ldots, Yk).$$

Here, the components are defined as:

$$X_j = \frac{\partial}{\partial x_i}, \quad i = 1, \ldots, m, \quad Y_j = |x|^{\gamma} \frac{\partial}{\partial y_j}, \quad j = 1, \ldots, k.$$

The Baouendi-Grushin operator on R^n is the operator

$$\Delta\gamma = \nabla\gamma \cdot \nabla\gamma = \Delta x + |x|^{2\gamma}\Delta y,$$

where Δx and Δy are Laplace operators in the variables x ∈ Rm and y ∈ Rk, respectively. In this setting, we have the following inequality:

$$\int_{\mathbb{R}^n} \left(|\nabla_x f|^2 + |x|^{2\gamma} |\nabla_y f|^2 \right) dz \geq \left(\frac{Q-2}{2} \right)^2 \int_{\mathbb{R}^n} \frac{|x|^{2\gamma}}{|x|^{2+2\gamma} + (1+\gamma)^2 |y|^2} |f|^2 dz,$$

for every f ∈ C∞0Rm × Rk\{(0, 0)} with Q = m + (1 + γ)k. Here, ∇xf represents the gradient of f with respect to the variable x, while ∇yf denotes the gradient of f with respect to the variable y.

In this paper, we obtain Hardy and Rellich identities related to Baouendi-Grushin operator by factorizing differential expressions, inspired by the work of Gesztesy and Littlejohn [2]. In the special cases, we are able to derive classical, critical and weighted Hardy inequalities in the Euclidean setting. Similarly, we recover the improved L2 - Caffareli-Kohn-Nirenberg inequality, which in turn gives the Heisenberg and Hydrogen Uncertainty Principles. Finally, under certain parameters one of our results yields known Hardy–Rellich-type inequalities, including the classical Rellich inequality and the Schmincke's one-parameter inequality.

Keywords: Hardy identities, Rellich identities, Baouendi-Grushin operator, factorization method.

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ON THE BEST CONSTANTS FOR INTEGRAL HARDY INEQUALITIES WITH ANY HOMOGENEOUS-QUASI NORM

It is well-known that within the L^p spaces with $0 < p < 1$, the Hardy inequality does not hold for arbitrary non-negative functions, yet it remains valid for non- negative monotone functions. In [1] Burenkov found sharp constants in integral Hardy- type inequalities for non-negative monotone functions when $0 < p < 1$.

In [2] it was introduced Hardy inequalities within metric measure spaces possessing polar decompositions when $p = 1$ and $1 q <$. The authors in [3] obtained weighted integral Hardy inequalities and conjugate integral Hardy inequalities on homogeneous Lie groups for any homogeneous quasi-norm with sharp constants within the range $1 < p q <$.

In this talk, we explore Hardy-type integral inequalities for $0 < p < 1$, incorporating a second parameter $q > 0$ with sharp constants. These inequalities represent novel generalizations of those obtained in [1].

This talk is based on the joint research with Nurgissa Yessirkegenov (SDU University and Institute of Mathematics and Mathematical Modeling, Kazakhstan).

Keywords: Integral Hardy-type inequality, sharp constant, homogeneous Lie groups.

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GENERALISED PICONE'S IDENTITY FOR Δ_γ–LAPLACE OPERATOR AND ITS APPLICATIONS

Let us recall the classical Picone's identity: for differentiable functions $v > 0$ and $u \geq 0$, we have

$$|\nabla u|^2 + \frac{u^2}{v^2}|\nabla v|^2 - 2\frac{u}{v}\nabla u \cdot \nabla v = |\nabla u|^2 - \nabla\left(\frac{u^2}{v}\right) \cdot \nabla v \geq 0. \qquad (1)$$

Applications of identity (1) to second-order elliptic equations and systems are enormous [2-4]. We discuss the generalized variable exponent of the anisotropic Δ_γ – Laplace operator for the nonlinear equivalent of Picone's identity. The generalized variable exponent Picone type identities developed by Abolarinwa A. [1]. Franchi

B. and Lanconelli E. discussed the Δ_γ -operator in [5]. Luyen developed a nonlinear analogue of the Picone's identity using this operator [6].

Additionally, we establish the Picone identity for the p-biharmonic operator L^2,

and provide applications to generic weighted Rellich type inequalities, a Hardy-type inequality, and the Sturmian comparison principle.

Key words: Laplace operator, Picone's identity.